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Monitoring for Temporal Changes in Soil Salinity using Electromagnetic Induction Techniques

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ABSTRACT

Electromagnetic induction surveys are often used in practice to estimate field-scale soil salinity patterns, and to infer changing salinity conditions with time. We developed a statistical monitoring strategy that uses electromagnetic induction data and repetitive soil sampling to measure changing soil salinity conditions. This monitoring approach requires (i) the estimation of a conditional regression model that is capable of predicting soil salinity from electromagnetic (EM) survey data, and (ii) the acquisition of new soil samples at two or more previously established survey sites, so that formal tests can be made on the differences between the predicted and observed salinity levels. We examined two test statistics in detail: a test for detecting dynamic spatial variation in the new salinity pattern and a test for detecting a change in the field median salinity level with time. We applied this monitoring and testing strategy to two EM survey–soil salinity data sets collected at multiple points in time from the saline irrigation district of Flumen, Spain. Our results demonstrate that this monitoring approach was successfully able to quantify the temporal changes in the soil salinity patterns occurring within these two fields.

The use of electromagnetic induction sensors for the assessment and monitoring of soil salinity conditions has received considerable attention in the soil science literature (Lesch et al., 1995a,b; Díaz and Herrero, 1992; Hendrickx et al., 1992; Rhoades, 1992; Rhoades and Corwin, 1990; Rhoades and Miyamoto, 1990; Slavich, 1990; Williams and Baker, 1982; McNeill, 1980). These sensors can generally be classified into one of three types: (i) four-electrode sensors, including either surface array or insertion probes, (ii) remote EM induction sensors, such as the Geonics EM-31, EM-34, and EM-38 (Geonics Ltd., Mississauga, ON)[1], and (iii) time domain reflectometric sensors (Rhoades, 1992; Dalton, 1992). Of these three sensor types, the remote EM instruments have received the most attention for field-scale agricultural applications, particularly the EM-38.

In 1992, Díaz and Herrero discussed the monitoring of soil salinity conditions with time in two fields using EM-38 survey data. In their study, both EM-38 and sample soil salinity data were collected at multiple points in time within each field. The focus of the study was to examine various ways that one might use the multistage EM-38 survey data to monitor and predict the time-dependent changes occurring in the field-scale soil salinity conditions. Their data sets are rather unique in that they represent one of the few published data sets where both EM-38 and soil salinity data have been acquired at multiple points in time from within the same fields.

A comprehensive statistical methodology for the prediction of soil salinity using EM signal data was suggested by Lesch et al. (1995a,b). This prediction approach was based on the development of field-specific multiple linear regression models that could be used to predict soil salinity levels from EM-38 survey data. These researchers also suggested that the regression modeling methodology could be employed to monitor changes in the soil salinity conditions with time, provided additional soil samples were acquired at one or more survey sites in the future.

There is a clear need for the development of cost-effective, quantitative salinity monitoring techniques. The initial diagnosis of the soil salinity conditions within a field typically represents just the first step in a long-term reclamation project or salinity management process. Periodic monitoring of the evolving salinity conditions is just as essential when identifying the most worthwhile reclamation or management strategies, and for developing meaningful cost–benefit analyses. Furthermore, this type of information is often needed for the calibration and testing of various types of dynamic salinity transport models.

Unfortunately, there appears to be no generally accepted, quantitative statistical monitoring strategy suggested in the soil science literature that can incorporate repetitive EM survey and soil sample data into some

[1] Mention of trademark or proprietary products in this manuscript does not constitute a guarantee or warranty of the product by the U.S. Department of Agriculture and does not imply its approval to the exclusion of other products that may also be suitable.

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Abbreviations: EM, electromagnetic; ANOVA, analysis of variance; EC_e, electical conductivity of the saturated soil extract; MLR, multiple linear regression; MSE, mean square error.

type of formal statistical "test" (for detecting a change in soil salinity conditions with time). For example, suppose that an EM survey is conducted across a given field, soil samples are acquired at some of these survey sites, and a regression model is estimated from this data. In this example, assume that the regression model can be used to convert the EM survey data into a predicted soil salinity level at each survey site across the field. Now, at some point in the future suppose one wishes to formally test for a change in the spatial salinity pattern or average salinity level in this field. Then how should this be done? Should one conduct a new EM survey, acquire new soil samples, and fit a new regression model? Or should one only acquire new soil samples without resurveying the field; or perhaps only conduct a new survey without collecting any new soil samples (and in either case, somehow use this data in conjunction with the old regression model). Additionally, does it matter if the two survey grids or sets of soil sample sites are collocated?

We examined these questions, and developed a coherent statistical monitoring methodology for use with the typical, regression-based EM survey techniques commonly applied in practice. We first developed an appropriate regression equation by incorporating the ideas behind the more traditional, mixed linear analysis of variance (ANOVA) model into the regression modeling assumptions. These modeling assumptions in turn determine how the overall survey should be carried out; i.e., when and where the EM survey and soil salinity data should be acquired. Next, we developed two statistical tests based on these modeling assumptions. The first test can be used to determine if the salinity pattern has changed in a spatially variable manner, and the second test can be used to determine if the average salinity level across the entire field has changed with time. We then used the previously mentioned survey data (Díaz and Herrero, 1992) to demonstrate this monitoring methodology, and used these statistical tests to quantify changes occurring in soil salinity patterns with time.

MATERIALS AND SURVEY METHODS

The data to be analyzed come from two 0.5-ha salt-affected parcels in the irrigation district of Flumen (Aragon, Spain). The first parcel (P, 0.54 ha) contains soil classified as a fine, mixed (calcareous), thermic Oxyaquic Torrifluvent. At the time of sampling, it was slightly leveled and had been regularly cropped with rice (*Oryza sativa* L.) duirng the previous 25 yr. The second parcel (M, 0.40 ha) is more texturally heterogeneous. Approximately 55% of the parcel contains soil classified as a coarse-silty, mixed (calcareous) Xeric Torriorthent. The remaining 45% of the parcel is classified as (i) a loamy, mixed (calcareous) thermic shallow Lithic Xeric Torriorthent (15%); (ii) a coarse-loamy, mixed (calcareous), thermic Xeric Torrifluvent (15%); and (iii) a coarse-loamy, mixed, thermic Xeric Haplocalcid (15%). In 1987 the parcel was laser-leveled and a buried tile drain system was installed. Thereafter, the M parcel was used for maize (*Zea mays* L.) production.

Electromagnetic induction surveys and soil sampling were carried out in both parcels in 1988 and 1989, and in the M parcel in 1990. Electrical conductivity readings were acquired at the soil surface across each field on a 10 by 10 m grid using a Geonics EM-38 meter. Both horizontal (EM_H) and vertical (EM_V) readings were acquired at each site, and then temperature corrected to 25°C using the correction coefficients given U.S. Salinity Laboratory Staff (1954). The P parcel was surveyed twice (March 1988 and January 1989), and the M parcel was surveyed three times (May 1988, January 1989, and April 1990).

A limited number of soil samples were acquired during each survey in both fields. The sampling sites that were selected were chosen to both (i) span the observed range in the EM-38 signal data, and (ii) provide reasonable (i.e., approximately uniform) coverage across each parcel. The numbers and locations of these sample sites varied from year to year (see Fig. 1). Soil samples at each sample site were acquired in 0.25-m intervals down to depths of 1.5 and 1.0 m in the P and M parcels, respectively. Both electrical conductivity of the saturated soil extract (EC_e) and electrical conductivity of the 1:5 soil water extract were determined using standard laboratory methods on each soil sample (U.S. Salinity Laboratory Staff, 1954).

Some additional information concerning the 1988 P parcel and 1988 and 1989 M parcel EM survey and soil salinity data can be found in Díaz and Herrero (1992) and López-Bruna and Herrero (1996). However, in the analysis that follows, we

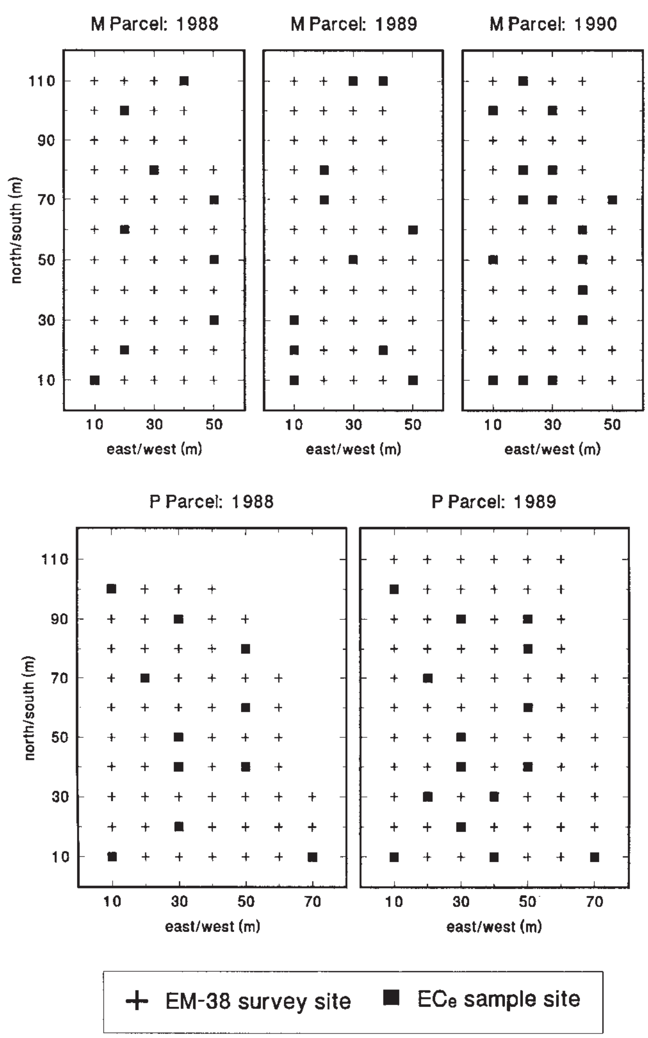

Fig. 1. Locations of EM-38 survey sites and soil extract electrical conductivity (EC_e) sample sites in parcels P and M for each survey date.

will consider only the EC_e data, since this data is generally considered a more reliable measurement of soil salinity. Additionally, to simplify the analysis, we have chosen to only relate the EM-38 signal data to the average of the EC_e data within the 0- to 1-m sampling depth in each field.

STATISTICAL THEORY

The prediction of soil EC_e from EM measurements requires that a model be developed that relates the two sets of data to each other. Numerous researchers have suggested various deterministic (theoretically based) or statistical EC_e–EM prediction models (López-Bruna and Herrero, 1996; Lesch et al., 1995a,b; Yates et al., 1993; Rhoades, 1992; Slavich, 1990; Williams and Baker, 1982; McNeill, 1980). The approach we used was to develop field-specific, multiple linear regression (MLR) models that can predict soil EC_e levels at each survey point from the acquired EM data (Lesch et al., 1995a). Therefore, a review of the modeling assumptions intrinsic to this regression approach will be given first. We will then briefly review the modeling assumptions behind the more traditional, mixed linear analysis of variance model. This is the most common type of statistical model one would typically use to test for changing salinity conditions if no EM covariate data were available. Finally, we will show how these ANOVA assumptions can be incorporated into a conditional regression model, and demonstrate how such a model can be used to test for a change in soil salinity with time.

Multiple Linear Regression Model Definitions and Assumptions

In the MLR modeling approach defined here, we assume that there exists a linear relationship between the natural log (ln) transformed soil EC_e levels and the ln-transformed EM readings. We also assume that additional "trend surface" parameters may need to be included in the MLR model to account for spatial drift or lateral trends in the EM signal data unrelated to the soil salinity levels. For example, when two EM-38 signal readings are acquired at each survey site, a MLR salinity prediction model with first-order trend surface parameters would be defined as

$$\ln(\mathrm{EC_e}) = b_0 + b_1 z_1 + b_2 z_2 + b_3 u + b_4 v + \epsilon \qquad [1a]$$

where the (u,v) variables represent the spatial coordinates of each survey site, z_1 and z_2 are defined as $z_1 = \ln(\mathrm{EM_V}) + \ln(\mathrm{EM_H})$, $z_2 = \ln(\mathrm{EM_V}) - \ln(\mathrm{EM_H})$, and ϵ represents the stochastic, residual error component. Note that the adding and subtracting of the EM signal components (inherent in the z_1 and z_2 definitions) is simply done to reduce the effects of signal multicollinearity (Myers, 1986).

It is generally more convenient to write Eq. [1a] in matrix notation. Suppose that there are salinity data from $i = 1, 2, ..., n$ sample sites, where these "calibration" sites form a subset of a larger set of N EM survey sites. Define $\mathbf{y} = [\ln(\mathrm{EC_{e1}}), \ln(\mathrm{EC_{e2}}), ..., \ln(\mathrm{EC_{en}})]^{\mathrm{T}}$, $\mathbf{x}_i = (1, z_{1i}, z_{2i}, u_i, v_i)^{\mathrm{T}}$, $\mathbf{X} = (x_1, x_2, ..., x_n)^{\mathrm{T}}$, $\boldsymbol{\beta} = (b_0, b_1, b_2, b_3, b_4)^{\mathrm{T}}$, and $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2, ..., \epsilon_n)^{\mathrm{T}}$, where T represents the matrix transpose symbol. Then Eq. [1a] can be expressed as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \qquad [1b]$$

Likewise, the predicted natural-log-transformed salinity at the ith survey site may be written as $\hat{y}_i = \mathbf{x}_i^{\mathrm{T}}\mathbf{b}$, where $\mathbf{b}$ represents the estimated $\boldsymbol{\beta}$ parameter vector. From standard regression theory, a $100(1 - \alpha)\%$ prediction interval for a $\ln(\mathrm{EC_e})$ sample acquired at this site would be $\hat{y}_i \pm t_{(\alpha/2,n-p-1)}\, s(1 + \mathbf{x}_i^{\mathrm{T}}(\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{x}_i)^{1/2}$, where t is the t-distribution, $p + 1$ is the number of estimated model parameters including the intercept, and s^2 is the estimated mean square error of the regression model (Myers, 1986).

In Eq. [1a] and [1b] the residuals, ϵ, are assumed to be normally distributed with homogeneous variance and spatially uncorrelated. All of these assumptions must be verified before the model can be used for prediction purposes. The assumption of residual spatial independence can be examined by using a Moran residual autocorrelation test (Brandsma and Ketellapper; 1979, Lesch et al., 1995a). The remaining residual assumptions should be verified using the standard residual diagnostic plots or tests (Myers, 1986; Atkinson, 1985; Weisberg, 1985).

A final assumption intrinsic to the regression modeling approach concerns the $\mathbf{X}$ matrix, which may be considered fixed or random. When the $\mathbf{X}$ matrix is considered fixed, one implicitly assumes that this matrix contains all the information about which any inference will be made. In practice, this means that one is actually restricting any inference (i.e., model predictions, statistical tests, etc.) to the EM survey grid, rather than the entire field. For example, this is the approach taken in Lesch et al. (1995a,b).

An alternative approach would be to consider the $\mathbf{X}$ matrix to be random, rather than fixed. By this we mean that the EM information associated with the finite number of survey sites observed during the survey process actually represent just a subsample of the potentially infinite number of survey sites that could have been observed. This type of regression equation is commonly referred to as a *conditional regression* model, since the model is conditional on the observed (random) $\mathbf{X}$ data matrix. Such a model usually arises when the covariate data (i.e., the $\mathbf{X}$ matrix) comes from either observational studies or designed experiments with both fixed and random treatment effects (Montgomery, 1984; Ryan, 1997). Under this approach it is still possible to treat the $\mathbf{X}$ matrix as though it were fixed, provided that conditional distribution of the observed response data, given the observed $\mathbf{X}$ matrix, is normal with constant variance, and that the distribution of $\mathbf{X}$ does not depend on either the $\boldsymbol{\beta}$ parameter vector or the error variance (Neter et al., 1989). However, the inference space in this approach could include the entire field, rather than being exclusively restricted to just the survey grid.

The ANOVA (Mixed Linear Model) Approach

To clarify how one can use a conditional regression equation for monitoring purposes, it will be helpful to first briefly review the modeling assumptions inherent in the more traditional, mixed linear ANOVA model. This is the model one would typically use if no EM covariate data were available; i.e., all monitoring activities were to be based only on the repetitive collection of soil samples.

For this discussion, define y_{ijk} as the observed natural-log-transformed salinity level from the jth sample site during the ith time frame, where $i = 1$ or 2. Suppose also that we acquire two salinity samples from each site (for example, by taking two soil cores arbitrarily close together), and let the $k = 1, 2$ subscript represent these "replicate samples." Then a traditional mixed linear model can then be written as

$$y_{ijk} = u + t_i + b_j + (tb)_{ij} + e_{ijk} \qquad [2a]$$

for $i = 1, 2$, $j = 1, 2, ..., n$, $k = 1, 2$, and with $b_j \sim$ iid $N(0, \sigma_b^2)$, $(tb)_{ij} \sim$ iid $N(0,\sigma_{tb}^2)$, and $e_{ijk} \sim$ iid $N(0,\sigma^2)$. Under these assumptions, it is well known (i.e., Montgomery, 1984) that the expected mean squared error for t_i, $(tb)_{ij}$, and e_{ijk} are $\mathrm{E\{MS}t\} = \sigma^2 + 2\sigma_{tb}^2 + 2n(t_1^2 + t_2^2)$, $\mathrm{E\{MS}tb\} = \sigma^2 + 2\sigma_{tb}^2$, and $\mathrm{E\{MS}e\} = \sigma^2$. Thus the ratio of $\mathrm{MS}tb/\mathrm{MS}e$ can be compared with an F

distribution to test for $\sigma_{tb}{}^2 = 0$, and the MSt/MStb ratio can be used to test for $t_1 - t_2 = 0$.

In a general ANOVA model, the e_{ijk} variance component typically represents sampling error, but the $(tb)_{ij}$ interaction variance component can at times be difficult to interpret. When analyzing spatial data, however, this interaction variance component has an obvious meaning. When $\sigma^2_{tb} > 0$, one can conclude that changes in the natural-log-transformed salinity levels are spatially variable (i.e., different from site to site). Hence, testing $\sigma^2_{tb} = 0$ is equivalent to testing for spatially dynamic change across the field, while the $t_1 - t_2 = 0$ test represents a test about the average shift with time in the mean natural-log-transformed salinity level across the entire field. We can therefore use these two statistical tests to determine which one of the following four scenarios seems most likely, given the observed data:

Scenario	*Corresponding hypothesis*
1. No change with time	$\sigma^2_{tb} = 0$ and $t_1 - t_2 = 0$
2. Static (spatially constant) change with time	$\sigma^2_{tb} = 0$ and $t_1 - t_2 \neq 0$
3. Dynamic (spatially variable) change with time (global shift not statistically significant)	$\sigma_{tb}{}^2 \neq 0$ and $t_1 - t_2 = 0$
4. Dynamic (spatially variable) change with time (global shift is statistically significant)	$\sigma^2_{tb} \neq 0$ and $t_1 - t_2 \neq 0$

When no replicate samples are available, then the two error components become confounded together and hence the ANOVA model becomes

$$y_{ij} = u + t_i + b_j + \varsigma_{ij} \qquad \varsigma_{ij} = (tb)_{ij} + e_{ij} \qquad [2b]$$

for $i = 1, 2$ and $j = 1, 2, ..., n$. In practice, when one computes a paired t-test, one is actually using Eq. [2b] shown above, but respecified as

$$y_{2j} - y_{1j} = d_j = d_0 + \boldsymbol{\eta}_j + e_{2j} - e_{1j} \qquad [3]$$

for $j = 1, 2, ..., n$. In Eq. [3], $d_0 = t_2 - t_1$, $\boldsymbol{\eta}_j \sim$ iid $N(0, \sigma^2_{tb})$, $e_{ij} \sim$ iid $N(0, \sigma^2)$, $\boldsymbol{\eta}_j$ and e_{ij} are assumed uncorrelated, and typically one assumes that $\sigma^2_{tb} = 0$.

The Conditional Regression Approach

It is possible to formulate a conditional regression model using the mixed linear modeling assumptions just described. Define y_{1j} and y_{2k} as the observed natural-log-transformed salinity levels from the jth and kth sample site acquired during the first and second time frames, where $j = 1, 2, ..., n$ and $k = 1, 2, ..., m$. Let $\mathbf{y}_1$ represent the vector of observations from the first time frame, and $\mathbf{y}_2$ represent the observations from the second time frame. Additionally, define $\mathbf{X}_1$ as the matrix (grid) of EM covariate signal data observed during the first time frame. For this discussion, suppose that a survey grid of size N ($N > n,m$) of representative EM covariate data has been acquired during the first time frame only, and that the n and m sample sites (from the first and second time frames, respectively) are chosen from this grid. Note that the two sets of sample sites need not be collocated. Furthermore, assume that the conditional distribution of $\mathbf{y}$ given the observed $\mathbf{X}_1$ matrix is normal with constant variance, and that the distribution of $\mathbf{X}_1$ does not depend on either the $\boldsymbol{\beta}$ parameter vector or the error variance. Assume that a suitable model for the first time frame is

$$\mathbf{y}_1 \mid \mathbf{X}_1 = \mathbf{X}_1\boldsymbol{\beta} + \boldsymbol{\epsilon}_1 \qquad [4]$$

where $\boldsymbol{\epsilon}_1 \sim N(0,\sigma\,\mathbf{I})$ and $\mathbf{I}$ represents the identity matrix. Furthermore, assume that a suitable model for the second time frame is

$$\mathbf{y}_2 \mid \mathbf{X}_1 = \mathbf{X}_1\boldsymbol{\beta} + \mathbf{d}_0 + \boldsymbol{\eta} + \boldsymbol{\epsilon}_2 \qquad [5]$$

where $\mathbf{d}_0 = [d_0, d_0, ..., d_0]$, $\boldsymbol{\eta} \sim N(\mathbf{0},\theta\mathbf{I})$, $\boldsymbol{\epsilon}_2 \sim N(0,\sigma\mathbf{I})$, and the $\boldsymbol{\eta}$, $\boldsymbol{\epsilon}_1$, and $\boldsymbol{\epsilon}_2$ random error components are mutually independent. Hence, $(\mathbf{y}_2 \mid \mathbf{X}_1) - (\mathbf{y}_1 \mid \mathbf{X}_1) = \mathbf{d}_0 + \boldsymbol{\eta} + \boldsymbol{\epsilon}_2 - \boldsymbol{\epsilon}_1$, which is simply Eq. [3] written in matrix format. Therefore, in Eq. [5], d_0 represents the shift in the average natural-log-transformed salinity level between the two time frames and the additional error term $\boldsymbol{\eta}$ represents the dynamic variability component.

After acquiring the first set of n calibration samples, suppose Eq. [4] is estimated as $\hat{\mathbf{y}}_1 \mid \mathbf{X}_1 = \mathbf{X}_n\,\mathbf{b}$, were $\hat{\mathbf{y}}_1$ represents the vector of predicted natrual-log-transformed salinity levels computed using Eq. [4], $\mathbf{X}_n$ represents the EM covariate matrix associated with the n sites, and $\mathbf{b}$ represents the estimated parameter vector. Then, from standard linear regression theory, the prediction error associated with a new set of k predicted sites located on the grid would be distributed as multivariate normal with a mean of 0 and a variance–covariance matrix of $\sigma^2(\mathbf{I}_k + \mathbf{H}_k)$, where $\mathbf{H}_k = \mathbf{X}_k^{\mathrm{T}}(\mathbf{X}_n^{\mathrm{T}}\mathbf{X}_n)^{-1}\mathbf{X}_k$, $\mathbf{X}_k$ represents the matrix of EM covariate data associated with the k prediction sites, and $\mathbf{X}_n$ is the matrix of covariate data associated with the n calibration sites (Graybill, 1976). In other words, $(\mathbf{y}_1 - \hat{\mathbf{y}}_1) \mid \mathbf{X}_1 \sim N[\mathbf{0},\sigma^2(\mathbf{I}_k + \mathbf{H}_k)]$. However, note that these prediction errors are only valid for new samples acquired during the first time frame; i.e., only for new samples acquired at the same time as the calibration samples.

Now, suppose m new samples located on the grid are acquired during the second time frame. Let $\mathbf{y}_2$ represent this vector of sample observations, $\hat{\mathbf{y}}_1$ represent the corresponding vector of predicted levels computed from Eq. [4] at these m sites, and define $\mathbf{H}_m = \mathbf{X}_m^{\mathrm{T}}(\mathbf{X}_n^{\mathrm{T}}\mathbf{X}_n)^{-1}\mathbf{X}_m$, where $\mathbf{X}_m$ represents the matrix of EM covariate data associated with these m prediction sites from the first time frame. Then Eq. [5] implies that the prediction error associated with these sites would be $(\mathbf{y}_2 - \hat{\mathbf{y}}_1) \mid \mathbf{X}_1 \sim N[\mathbf{d}_0, \theta\,\mathbf{I}_m + \sigma^2(\mathbf{I}_m + \mathbf{H}_m)]$; i.e., the observed (Time 2) minus predicted (Time 1) differences will contain two sources of error, and may no longer be equal (on the average) to 0.

Note that the vector of differences $\mathbf{d} = \mathbf{y}_2 - \hat{\mathbf{y}}_1$ is observable (once the samples from the second time frame have been acquired), and that under our modeling assumptions, its distribution is known. Furthermore, $\mathbf{d}$ implicitly contains information about d_0 and θ^2. Hence estimates and tests concerning both d_0 and θ^2 are derivable from these observed differences. To motivate these derivations, assume that Eq. [4] has been estimated as $\hat{\mathbf{y}}_1 \mid \mathbf{X}_1 = \mathbf{X}_n\mathbf{b}$, and let s^2 represent the calculated model mean square error with $n - p - 1$ degrees of freedom. Additionally, suppose that $\mathbf{d} = \mathbf{y}_2 - \hat{\mathbf{y}}_1$ has been observed, where the vector $\mathbf{d} = \{d_1, d_2, ..., d_m\}$. Define the calculated sample mean and variance of these observed differences as $\overline{u}$ and w^2, where $\overline{u} = (1/m)(d_1 + d_2 + ... + d_m)$ and $w^2 = [1/(m - 1)][(d_1 - \overline{u})^2 + (d_2 - \overline{u})^2 + ... + (d_m - \overline{u})^2]$. Clearly $\overline{u}$ represents a conditionally unbiased estimate of d_0. Furthermore, given the previously stated modeling assumptions, the following three results can be derived: (i) an F test for determining if $\theta^2 > 0$, (ii) a method of moments estimate of θ^2, and (iii) an approximate t-test for determining if $d_0 = 0$. These results are given below:

1. An F test for determining if $\theta^2 > 0$ can be computed as $\phi = (\mathbf{d} - \overline{\mathbf{u}})^{\mathrm{T}}\Sigma^{-1}(\mathbf{d} - \overline{\mathbf{u}})/(m - 1)s^2$, where $\Sigma = (\mathbf{I} + \mathbf{H}_m)$, and where ϕ is compared to an F distribution with $m - 1$ and $n - p - 1$ degrees of freedom.

2. The expected value of w^2 is $\theta^2 + \sigma^2(1 + \lambda_1 - \lambda_2)$, with $\lambda_1 = (1/m)\Sigma\, h_{ii}$ and $\lambda_2 = \{1/[m(m - 1)]\}\Sigma\Sigma\, h_{ij} \vee i \neq j$ (where h_{ij} represent the ith, jth diagonal element of the $\mathbf{H}_m$ matrix). Hence, a method of moments estimate of θ^2 is $v^2 = w^2 - s^2(1 + \lambda_1 - \lambda_2)$.
3. An approximate t-test for $d_0 = 0$ can be computed as $c = \bar{u}/g$, where $g^2 = (1/m)v^2 + 2s^2[(1/m) + h_{mu}]$, $h_{mu} = \mathbf{x}_{mu}^{\mathrm{T}}(\mathbf{X}_n^{\mathrm{T}}\mathbf{X}_n)^{-1}\mathbf{x}_{mu}$, $\mathbf{x}_{mu} = (1/m)(\mathbf{x}_1 + \mathbf{x}_2 + \ldots \mathbf{x}_m)$, and where c is compared with a t distribution with $n - p - 1$ degrees of freedom. Note that this test statistic assumes that the two sets of soil samples are not collocated.

Proofs for each of these derivations are shown in the Appendix.

Lesch et al. (1995a) suggested a mean shift test statistic defined as $\bar{u}/\tau$, where $\tau^2 = s^2[(1/m) + h_{mu}]$, and the remaining terms are the same as those shown in Result 4. Under our modeling assumptions, this would *not* be a valid test for determining if $d_0 = 0$, since the variance term is incorrectly specified. What this statistic actually tests is only whether there is a statistically significant difference between the observed and predicted mean natural-log-transformed salinity level across the m new monitoring sites; it is not a valid test for inferring change across the entire field.

Some other important features about this conditional regression model are worth highlighting. First, this model assumes that there are two potential sources of error present during the second time frame; sampling error (which is also present during the first time frame) and dynamic spatial variation. Hence, in order to test for a change in the mean salinity level with time, both errors must be accounted for. This means that both variance components must be estimated, *which in turn means that we must acquire soil samples during both time frames*. Second, we do not have to acquire a new grid of EM-38 survey data during the second time frame to compute these test statistics. Acquiring a second set of survey data is almost always a good idea, however, because one can then estimate a new regression model (when necessary), which in turn can be used to estimate a new salinity map. This can prove to be very important, since Eq. [5] cannot be used to estimate the spatial salinity pattern during the second time frame, unless there is no dynamic salinity variation (i.e., unless $\theta^2 = 0$).

Third, under this conditional regression model, θ^2 (the dynamic salinity variance component) can be estimated even though only one sample is acquired during each time frame at each site. Hence, this approach will yield more information than the traditional paired t-test design, since in the latter design the treatment-block interaction error and the sampling error are confounded together. Additionally, the two sets of soil samples no longer need to be collocated. (This is a distinct advantage if the coring operation used to acquire the soil samples also disturbs the surrounding soil. For example, improperly filled bore holes at the first-stage sampling sites can become preferential pathways for vertical water movement during subsequent irrigations. Hence, one might want to avoid these sites during future monitoring activities.) Both of these features result from the fact that the blocking parameters in Eq. [3] have been replaced by regression parameters in Eq. [4] and [5].

Fourth, it is critically important that both sets of soil samples are associated with the same larger set of N survey sites collected during the first time frame. This requirement must be satisfied because the conditional regression model implicitly assumes that the $\mathbf{X}_1$ matrix is known (i.e., observed without error). If the $\mathbf{X}_1$ matrix was either unknown or had to be estimated, then we could not use it to develop a valid regression equation. In practice, this means we are restricted to sampling on the original grid, since these are the only sites where the EM-38 signal levels are known a priori.

Finally, although the $\mathrm{EC_e}$–EM regression relationship is typically modeled on the ln–ln scale, it is generally desirable to "back-transform" the predicted change in the average natural-log-transformed salinity level with time to a more meaningful estimate. In the conditional regression model, if we define $\hat{y}$ to represent a predicted mean natural-log-transformed salinity level, then $\exp(\hat{y})$ represents an unbiased estimate of the corresponding median salinity level. Since $\bar{u}$ is an estimate of $t_2 - t_1$, where t_1 and t_2 represent the field mean natural-log-transformed salinity levels at Times 1 and 2, $\exp(\bar{u}) = \exp(t_2)/\exp(t_1)$ and therefore a test of $\bar{u} = 0$ is equivalent to a test of $\exp(t_2)/\exp(t_1) = 1$. Hence, $100[\exp(\bar{u}) - 1]$ represents an estimate of the percentage increase (or decrease) in the field's median level with time. Likewise, the test of $\theta^2 = 0$ actually tests whether the salinity pattern in the field has changed in a strictly proportional manner (i.e., constant change on a percentage basis).

RESULTS

Preliminary Data Analysis

The EM survey and soil sampling grids for both parcels are shown in Fig. 1. Note that the number of survey points increased in the P parcel in 1989 (from 59 to 73 sites), while the survey grid remained relatively constant

Table 1. Distribution summary statistics for EM-38 survey and soil salinity data, by parcel and sampling date.

						Quantile estimates				
Data	Parcel	Date	N	Mean	Standard deviation	min.	q25	q50	q75	max.
				dS/min						
$\mathrm{EM_V}$	M	1988	52	1.42	0.33	0.83	1.19	1.37	1.59	2.50
	M	1989	50	1.47	0.35	0.72	1.29	1.44	1.58	2.66
	M	1990	51	1.11	0.28	0.51	0.94	1.07	1.32	1.92
	P	1988	59	3.01	0.66	1.48	2.52	3.12	3.49	4.75
	P	1989	73	3.39	0.87	1.77	2.74	3.47	3.95	5.40
$\mathrm{EM_H}$	M	1988	52	1.46	0.36	0.77	1.19	1.43	1.66	2.38
	M	1989	50	1.65	0.41	0.86	1.36	1.58	2.01	2.80
	M	1990	51	1.08	0.29	0.43	0.89	1.03	1.26	1.80
	P	1988	59	2.25	0.52	1.11	1.85	2.30	2.67	3.34
	P	1989	73	2.70	0.72	1.38	2.09	2.66	3.06	4.76
$\mathrm{EC_e}$	M	1988	9	8.29	3.07	3.68	6.73	10.4	10.7	11.2
	M	1989	11	7.31	2.49	2.46	5.95	7.93	8.90	10.7
	M	1990	16	6.70	2.06	1.23	5.77	6.49	8.18	10.5
	P	1988	11	5.95	2.66	2.50	3.72	6.28	7.37	10.2
	P	1989	15	6.96	2.88	2.30	4.33	6.89	7.92	13.6

in the M parcel for all 3 yr (52, 50, and 51 sites, respectively, for 1988, 1989, and 1990). The number of calibration sample sites also increased in both parcels for successive survey dates. A total of nine (1988), 11 (1989), and 16 (1990) sites were sampled in the M parcel, while 11 (1988) and 15 (1989) sites were sampled in the P parcel. Table 1 summarizes the EM-38 and EC_e data acquired in both parcels for all the sampling dates.

For model calibration purposes, we have chosen to average the EC_e data associated with the first four sampling depths into a single 0- to 100-cm salinity reading at each site. (The last two sample depths from each site in the P parcel will not be considered further.) Additionally, because the 1989 P parcel survey and 1990 M parcel survey contain the most sample sites, we have chosen to perform the conditional MLR modeling approach on these years. Hence, the 1988 P parcel and 1988 and 1989 M parcel salinity data will be used for testing purposes.

Figure 2 displays the 0- to 100-cm natural-log-transformed salinity levels plotted against the $\ln(EM_V) + \ln(EM_H)$ signal levels for both parcels. In the 1989 P parcel, the salinity–signal relationship appears reasonably linear. However, the same cannot be said for the 1990 M parcel data. The site labeled "m-16" appears extremely far removed from the other data values, and the pattern inherent in the remaining data is not well defined. In regression modeling jargon, site m-16 is referred to as a *high-leverage point* (Myers, 1986; Weisberg, 1985). High-leverage points can have an extreme influence on the estimation of a regression model. Hence, these points are often temporarily removed from the data during the initial model fitting process in order to avoid biasing the model selection process or parameter estimates.

Regression Modeling Results

Equation [1a] was specified as the initial model for both the M and P parcel EM–EC_e data. All 15 salinity samples from the 1989 survey were used in the P parcel model, while 15 of the 16 1990 samples were used in the initial M parcel model (site m-16 was temporarily set aside). The modeling results for each data set are discussed below.

In the initial P parcel model, neither the z_2 $[\ln(EM_V) - \ln(EM_H)]$ or u (east–west spatial coordinate) parameter

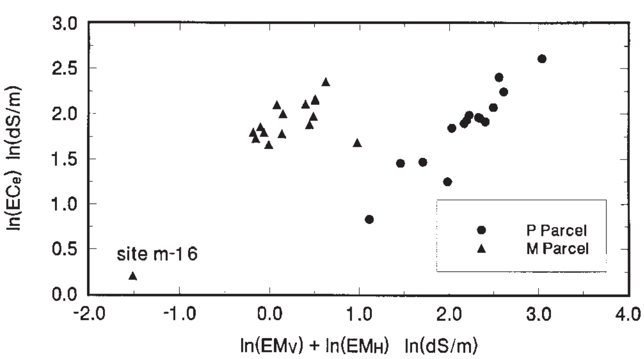

Fig. 2. Plot of $\ln(EC_e)$ sample data against z_1 $[\ln(EM_V) + \ln(EM_H)]$ signal data; note the high leverage point labeled m-16 in the parcel M data set.

estimates were found to be statistically significant. Hence, the revised P parcel model was defined to be

$$\ln(EC_e) \mid Z = b_0 + b_1 z_1 + b_2 v + \epsilon \quad [6]$$

The R^2 and mean square error (MSE) estimates for this model were 0.917 and 0.0196, respectively. The residuals appeared normally distributed with homogeneous variance, and the Moran spatial autocorrelation test statistic was nonsignificant.

In the initial M parcel model, all parameters were found to be statistically significant. However, a plot of the model residuals against the z_1 $[\ln(EM_V) + \ln(EM_H)]$ signal data revealed a strong curvilinear relationship. This suggested that a z_1^2 quadratic term should be included in the regression equation. Hence, the revised M parcel model was defined to be

$$\ln(EC_e) \mid Z = b_0 + b_1 z_1 + b_{11} z_1^2 + b_2 z_2 + b_3 u + b_4 v + \epsilon \quad [7]$$

The R^2 and MSE estimates for this model were 0.884 and 0.0075, respectively, and the new residuals displayed no additional assumption violations. Since Eq. [7] appeared reasonable, it was then reestimated using all 16 1990 salinity samples (including site m-16). The inclusion of site m-16 did not significantly change any of the parameter estimates, although it did affect some of the model statistics. The R^2 and MSE estimates for this model were 0.980 and 0.0069, respectively. Note that the large increase in the R^2 value is due entirely to the inclusion of site m-16; the initial R^2 value of 0.884 is more representative of the actual percentage of explained variability with respect to the majority of the M parcel salinity data.

The regression model summary statistics and parameter estimates are shown in Table 2 for both calibration models (Eq. [6] and [7]). Figure 3 shows a realistic assessment of the prediction accuracy for both models. In Fig. 3, the predicted salinity levels represent "jackknifed" predictions; i.e., each observation was sequentially removed from the regression model and then predicted using the remaining sample data (Myers, 1986). Figure 3 represents a good example of the prediction accuracy that can be obtained when the regression models are properly specified.

Table 2. Multiple linear regression model summary statistics and parameter estimates for the data from parcels P and M (Eq. [6] and [7]).

	P parcel, 1989	M parcel, 1990 (site m-16 excluded)	M parcel, 1990 (site m-16 included)
R^2	0.917	0.884	0.980
MSE	0.0196	0.0075	0.0069
Model F test	66.26	13.79	97.00
$P > F$	0.0001	0.0005	0.0001
		Parameter estimates	
b_0: intercept	0.192 (0.20)†	1.525 (0.10)	1.515 (0.09)
b_1: z_1	0.830 (0.08)	0.663 (0.15)	0.607 (0.06)
b_{11}: z_1^2	n/a‡	−0.455 (0.23)	−0.369 (0.04)
b_2: z_2	n/a	0.382 (0.12)	0.401 (0.11)
b_3: u	n/a	0.673 (0.27)	0.706 (0.24)
b_4: v	−0.297 (0.12)	0.242 (0.08)	0.244 (0.08)

† Standard errors in parentheses.
‡ Not applicable. Parameter was not included in the regression model.

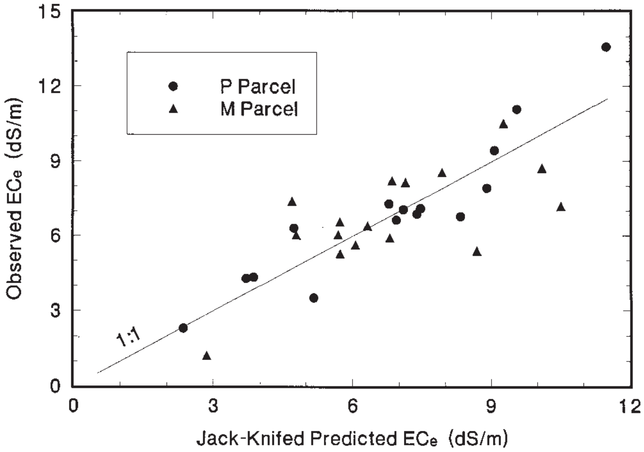

Fig. 3. Plot of the observed vs. jack-knifed predicted soil extract electrical conductivy (EC_e) data from the conditional regression models associated with parcels P and M.

Figure 4 displays the estimated median spatial salinity maps for parcels P and M, respectively. Ordinary kriging was used to interpolate the $\hat{y}$ regression predictions onto a 2.5-m grid before producing each map. In the P parcel, the salinity levels appear highest in the southwest corner of the field, and fall off rapidly in the northeast direction. The spatial salinity pattern in the M parcel is slightly more complicated. High salinity levels are apparent in both the northern and southern parts of the field, and lower levels occur along the eastern side of the field.

Testing for Changes in Field Median Salinity Levels with Time

Once new soil samples are acquired, the regression models derived for each parcel can be used to test for changes in both the field-scale natural-log-transformed salinity pattern and the mean field $\ln(EC_e)$ level with time. As described above, this can be done by testing if $\theta^2 = 0$ and $\bar{u} = 0$ using the formulas shown above (Results 1 and 3). On the back-transformed scale (in decisiemens per meter), the first test is equivalent to testing for strictly proportional change in the salinity pattern with time, while the second represents a test for a shift in the overall median field salinity level with time.

In this analysis, we cannot test for a forward shift, since the most recent survey data was used to develop the regression models. However, we can test for a backward shift. Thus, we can still test whether the spatial salinity pattern or median field salinity level appeared to change from 1988 to 1989 in the P parcel, and from either 1988 or 1989 to 1990 in the M parcel.

Table 3 presents the results for both of these tests. In the P parcel there is strong evidence that the observed change in the salinity pattern over 1988 to 1989 was spatially dynamic ($\phi = 8.82$, F test probability level $= 0.0004$, $v^2 = 0.156$). Additionally, the average observed $\ln(EC_e)$ levels for the 11 sample sites acquired in 1988 was 1.6854 ln(dS/m), while the average predicted $\ln(EC_e)$ levels for these same sites was 1.8823 ln(dS/m). This resulted in a $\bar{u}$ estimate of -0.1969, which was not found to be statistically significant using the approximate t-test ($\bar{u}/g = -1.38$, t-test probability level $= 0.194$). Hence, while there is strong evidence to suggest that the spatial salinity pattern was changing with time in the P parcel, there is not sufficient evidence to suggest that the overall field median salinity level had significantly increased.

For the M parcel, there is strong evidence that the observed changes in the salinity pattern from both 1988 to 1990 and 1989 to 1990 were also spatially dynamic ($\phi = 13.48, 5.81$; F test probability levels $= 0.0002, 0.0051$; $v^2 = 0.158, 0.058$, respectively). The estimated average natural-log-transformed salinity differences for each of these time frames were $\bar{u} = 0.3098$ and 0.1256, which had approximate t-test significance levels of 0.055 and 0.176, respectively. Thus, the 1988 to 1990 average natural-log-transformed difference appears to be significant at about the 0.05 level, while the 1989 to 1990 difference does not appear to be statistically significant.

Interpreting the Test Results

To infer any useful information from these test results, one must understand their interpretive value. As discussed above, when we test if $\theta^2 = 0$ what we are

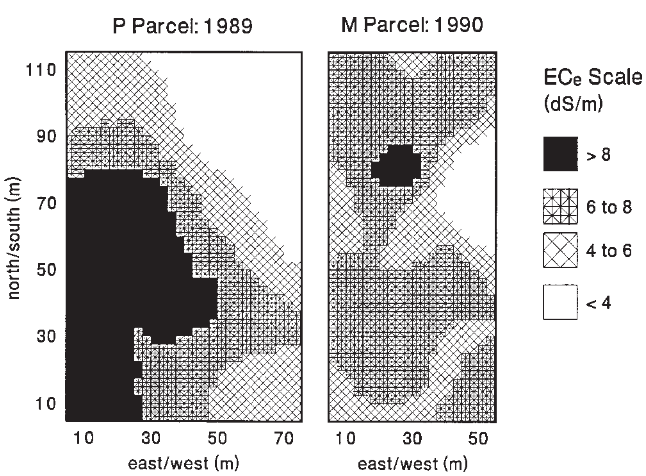

Fig. 4. Maps of the final, predicted median spatial salinity patterns in (a) the P parcel during 1989, and (b) the M parcel during 1990.

Table 3. Test results for $\theta^2 = 0$ (dynamic salinity variation) and $\bar{u} = 0$ (median shift) in parcels P and M.

	P, 1989–1988	M, 1990–1989	M, 1990–1988
	Estimates and test statistics for $\theta^2 = 0$		
ϕ	8.816	5.806	13.478
$P(\phi > F)$	0.0004	0.0051	0.0002
w^2	0.1786	0.0676	0.1684
s^2	0.0196	0.0069	0.0069
λ_1	0.1949	0.4322	0.4845
λ_2	0.0559	0.0469	0.0349
v^2	0.1563	0.0581	0.1585
	Estimates and test statistics for $\bar{u} = 0$		
$\bar{u}$	−0.1969	0.1256	0.3098
h_{mu}	0.0685	0.0819	0.0848
g^2	0.0205	0.0077	0.0203
c	−1.376	1.436	2.174
$P(c > t)$	0.1939	0.1766	0.0548
flux†, %	21.8	−11.8	−26.6

† Flux = the estimated percentage change in the field median salinity level; i.e., $100[\exp(-\bar{u}) - 1]$.

actually doing is testing whether the percentage change in the soil salinity level is constant across all m monitoring sites, given the estimated sampling variability (i.e., testing for strict proportional change in the absolute salinity levels). If there is either (i) no change or (ii) a proportional change, then this test should be nonsignificant. One the other hand, if the percentage change in soil salinity varies from site to site, then the F test should appear statistically significant. In this case we can conclude that there is "dynamic spatial variation," which is another way of saying that the spatial soil salinity pattern has not remained in equilibrium across the two time frames.

Likewise, when we test for $\bar{u} = 0$ we are actually testing if there is sufficient evidence to conclude that the overall, median field level has shifted up or down between the two time frames. When this test is rejected, but the test of $\theta^2 = 0$ is not rejected, then we can conclude that the spatial salinity pattern across the whole field has risen or fallen in a strictly proportional manner. On the other hand, if we cannot reject $\bar{u} = 0$, but do reject $\theta^2 = 0$, then we can conclude that there does appear to be dynamic spatial change in the salinity pattern, but that when averaged across the entire field, the overall change in the median salinity levels across the two time frames is not statistically significant.

Additionally, it is helpful to back-transform the $\bar{u}$ differences to equivalent percentage change in median salinity estimates. When sample salinity data has been acquired in the future, the proper back-transformation formula is $100[\exp(\bar{u}) - 1]$, where $\bar{u}$ represents the difference in the observed and predicted average $\ln(EC_e)$ levels. Since in this analysis the tests are being made into the past, the sign of the $\bar{u}$ estimate in the back-transformation formula should be reversed, yielding $100[\exp(-\bar{u}) - 1]$. For example, in the P parcel $-\bar{u} = 0.1969$; hence $100[\exp(0.1969) - 1] \approx 21.8\%$ and therefore the increase in the median salinity level from 1988 to 1989 was estimated to be about 22%. Likewise, in the M parcel the $-\bar{u}$ differences were -0.1256 and -0.3098, which translate to approximately -11.8% and -26.6% changes (i.e., a 11.8% and 26.6% decrease in the field median salinity level with time).

Given the above discussion, one can interpret the test results for the P and M parcels as follows. In the P parcel, there is clear evidence that the spatial soil salinity pattern is not in equilibrium. However, the estimated 22% increase in the median salinity level cannot be judged to be statistically significant. Potential reasons for the apparent dynamic variation could include changes with time in the manner in which this field is being managed or changing conditions in the soil physical or hydrological properties themselves. Therefore, in this field the analyst should try to identify (if possible) these various effects, and continue the monitoring program.

In the M parcel, the test results are more obvious. There is clear evidence that the spatial salinity pattern has been changing during a 2-yr time frame. The 26.6% decrease in the median salinity level between 1988 and 1990 is statistically significant, and the 1989 to 1990 difference, while not significant, was also found to be decreasing. Additionally, the magnitude of the 1988 to 1990 dynamic variation estimate is three times greater than the 1989 to 1990 estimate (which implies that the change in the spatial pattern duirng the 2 yr has been much greater than the apparent change in just 1 yr). All of these test results suggest that the spatial salinity pattern has been impacted by the tile lines (installed in 1987). In this case it would appear that the effect of the tile lines has been twofold: an overall lowering of the median salinity level across the field, and additionally, a dynamic redistribution of the spatial salinity pattern within the field.

Estimating a New Regression Model when Dynamic Spatial Variation is Present

As noted above, when dynamic spatial variation in the soil salinity pattern is detected, then a new regression model must be estimated in order to produce a new salinity map. In these situations, the new model should be estimated using the new EM-38 survey and soil salinity data (i.e., the survey and sample data obtained during the second time frame). The predicted map created from this second model can then be qualitatively compared with the predicted map from the first model in order to ascertain where the dynamic variation is occurring.

The test results discussed above suggest that new regression models should be estimated in both the P and M parcels. However, to conserve space, we have elected to only discuss the P parcel survey data here. As for the 1989 data, Eq. [1a] was initially used to model the 1988 P parcel survey and salinity data; these new regression model summary statistics and parameter estimates are given in Table 4. In the 1988 model all five parameter estimates were found to be statistically significant, and also quite different from the 1989 parameter estimates shown in Table 2.

The predicted natural-log-transformed salinity levels were then calculated across the 1988 P parcel EM survey grid using the fitted regression model in Table 4, and interpolated onto a 2.5-m grid to produce the 1988 median salinity map. Both the 1988 and 1989 maps are displayed in Fig. 5; note that part of the 1988 map has not been estimated due to missing survey sites (see Fig. 1). A qualitative comparison of these two maps suggests that there may have been a pronounced rise in soil

Table 4. Multiple linear regression model summary statistics and parameter estimates for the 1988 P parcel data (using Eq. [1a]).

	P parcel, 1988
R^2	0.970
MSE	0.0111
Model F test	49.01
$P > F$	0.0001
Parameter estimates	
b_0: intercept	2.539 (0.31)†
b_1: z_1	0.244 (0.09)
b_2: z_2	1.974 (0.37)
b_3: u	−1.334 (0.30)
b_4: v	−0.544 (0.12)

† Standard errors in parentheses.

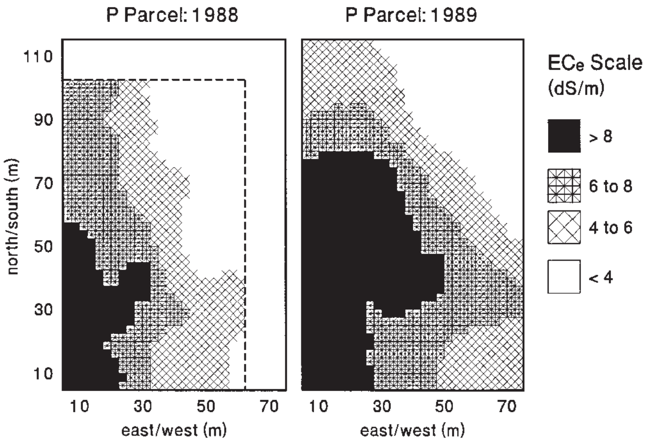

Fig. 5. Maps of the predicted median soil salinity levels in 1988 and 1989 within the P parcel; note the change in the spatial salinity pattern between the two years.

salinity in the southwest section of this field (but relatively little change across the rest of the field). This would explain the significant site-to-site variation. This also suggests that additional information is needed about this part of the field to determine why such an increase in soil salinity might be occurring.

DISCUSSION

As mentioned, there are different ways in which one might use repetitive EM-38 survey data to infer a change in soil salinity conditions with time. The monitoring strategy we have discussed represents a quantitative approach to this inference procedure. This approach requires at least one complete EM survey and two sets of soil samples, where both sets of soil samples are selected from the original EM survey grid. These requirements stem from the assumptions incorporated into the regression model. However, understanding these assumptions allows one to contrast this approach with other potential monitoring situations that could occur in practice.

There are two other approaches that seem intuitively reasonable. The first would be to collect two collocated EM-38 surveys (one survey during each time frame), but only collect the calibration soil samples during the first time frame. The idea behind this approach would obviously be to minimize the soil sampling requirements, while retaining the potential to detect change (through the second-stage EM signal data). With respect to our modeling assumptions, this approach is equivalent to assuming the following regression model relationship:

$$\mathbf{y}_2 \mid \mathbf{X}_1 = \mathbf{X}_1\boldsymbol{\beta} + d_0 + \boldsymbol{\eta} + \boldsymbol{\epsilon}_2 = \mathbf{y}_2 \mid \mathbf{X}_2 = \mathbf{X}_2\boldsymbol{\beta} + \boldsymbol{\epsilon}_2$$

In other words, the original regression model still holds exactly (both the model structure and the parameter estimates do not change); and hence any global shift or dynamic variation can be completely explained by the change in the second-stage EM signal data alone. Unfortunately, this assumption will often be violated in practice, since most EM–EC_e regression models tend to be both field and time dependent. Many temporal factors can contribute to these dependencies, including variations in the soil composition, texture, moisture, and temperature, changes in the physical bed–furrow structure, errors in instrument calibration or instrument-to-instrument variation, gross redistribution of the soil salinity throughout the soil profile, etc. Typically, it is not possible to account for all of these different effects using the same empirical equation with time, and hence the need for a new model. Note that when a new regression model is specified, the new relationship is assumed to be

$$\mathbf{y}_2 \mid \mathbf{X}_1 = \mathbf{X}_1\boldsymbol{\beta} + d_0 + \boldsymbol{\eta} + \boldsymbol{\epsilon}_2 = \mathbf{y}_2 \mid \mathbf{X}_2 = \mathbf{X}_2\boldsymbol{\Gamma} + \boldsymbol{\epsilon}_2$$

Hence, now the global shift and dynamic variation can be explained by a combination of the change in the EM signal *and* a change in the regression model structure or parameter estimates.

Occasionally, the assumption of a constant regression model with time may prove to be approximately true. However, one should not assume this unless it can be verified. Obviously, such an assumption cannot be tested unless new soil samples are acquired during the second survey, which puts the analyst back where he or she first started (i.e., having to collect new samples to prove that new soil samples are not needed!). Furthermore, even when this assumption is true, the test statistics developed here still cannot be employed without collecting the additional soil samples. Therefore, we do not recommend using this type of monitoring strategy.

The other approach that could be used in practice would be to collect two sets of EM surveys along with two sets of soil samples, but to collect the two surveys on nonoverlapping grids. Under our stated modeling assumptions, the tests suggested here cannot be computed for this type of survey data. This is true because the two grids do not overlap, and therefore there is no way to make a direct comparison between the two sets of sample data through either regression equation. Formally speaking, since the grids don't overlap, one cannot specify a model for $\mathbf{y}_2 \mid \mathbf{X}_1$, because the $\mathbf{X}_1$ matrix is unknown at each and every new (second-stage) sample location.

In order to analyze such data, additional statistical assumptions have to be made about the spatial distributions of both the EM survey and soil salinity data. In theory, data from this type of survey could be analyzed using some type of spatial–temporal cokriging model (Journel and Huijbregts, 1978; Cressie, 1991). However, a significant number of soil salinity samples would have to be collected during each type frame to estimate this type of model. Since it is clearly much more cost effective to try to minimize the soil sampling requirements, a reasonable solution to this problem is to simply employ collocated grids during the two surveys.

Two other facts are worth reiterating. As already mentioned, the test statistics described here require only one complete EM survey and two sets of soil samples. If dynamic salinity variation is detected, however, then a new regression model can only be estimated if a new set of EM signal data has been acquired (during the second time frame). In practice, it will typically be reasonable to expect dynamic variation to occur after a significant change in one or more management practices. (This is often why the monitoring is undertaken in the first place.) Therefore, one will generally need to perform a new EM survey when the second set of soil samples are acquired. Second, during each stage of the survey process, one should attempt to select calibration and monitoring sites that are representative of the full range of soil salinity variation within the field. Some suggestions on how to optimize the locations of both the calibration and monitoring sites can be found in Lesch et al. (1995b).

CONCLUSION

We have developed a statistical monitoring strategy for quantifying the change in field-scale salinity conditions with time. This monitoring strategy depends on (i) the estimation of a conditional regression model that is capable of predicting soil salinity from EM survey data, and (ii) the acquisition of new soil samples at two or more previously established survey sites, so that formal tests can be made on the differences between the predicted and observed salinity levels. Two test statistics have been described in detail: a test for detecting dynamic spatial variation in the new salinity pattern and a test for detecting a change in the field median salinity level with time. Both of these tests have been derived by incorporating the assumptions made in the traditional, mixed linear ANOVA model into the conditional regression model. The application and interpretation of this monitoring and testing strategy has been demonstrated using two EM survey–soil salinity data sets collected at multiple points in time from within the saline irrigation district of Flumen, Spain.

When a second EM survey is acquired along with the new soil samples, additional qualitative information can be gained about any detected change in the spatial salinity pattern. This new EM survey data can be used (with the new sample data) to estimate a new regression model, which in turn can be used to create a map of the new spatial salinity pattern. Although the second set of EM survey data is not required to compute the test statistics, this data must still be acquired in order to create a new salinity map (if such a map is desired). Therefore, we recommend that the analyst collect EM survey data and soil samples during both time frames.

As discussed above, both sets of soil samples must be chosen from the same larger set of EM survey sites collected during the first time frame. When this does not occur, the monitoring strategy we have described cannot be used because the test statistics cannot be computed. Hence, when two EM surveys are to be conducted, we strongly recommend that the survey grids be collocated. At the very minimum, all of the sample sites on the second EM survey grid must be collocated with previously determined survey sites (from the first EM survey) in order to compute the test statistics.

Finally, we do not recommend using this approach if gross spatial autocorrelation is detected in the residuals associated with the conditional regression model. Significant spatial autocorrelation in the residuals will result in biased regression parameter and variance estimates, and will adversely affect the two test statistics described here to an unknown degree.

APPENDIX

1. An F test for determining if $\theta^2 > 0$ can be computed as $\phi = (\mathbf{d} - \bar{\mathbf{u}})^{\mathrm{T}}\Sigma^{-1}(\mathbf{d} - \bar{\mathbf{u}})/(m-1)s^2$, where $\Sigma = (\mathbf{I} + \mathbf{H}_m)$, and then comparing ϕ to an F distribution with $m - 1$ and $n - p - 1$ degrees of freedom.

Proof: Under the modeling assumptions, if $\theta^2 = 0$ and d_0 is known, then $(\mathbf{d} - \mathbf{d}_0)^{\mathrm{T}}\Sigma^{-1}(\mathbf{d} - \mathbf{d}_0)/\sigma^2$ has a chi-square distribution with m degrees of freedom (Lieberman, 1961). Hence, $(\mathbf{d} - \bar{\mathbf{u}})^{\mathrm{T}}\Sigma^{-1}(\mathbf{d} - \bar{\mathbf{u}})/\sigma^2$ is distributed as a chi-square random variable with $m - 1$ degrees of freedom. Likewise, $(n - p - 1)s^2/\sigma^2$ has a chi-square distribution with $n - p - 1$ degrees of freedom, and is independent of $(\mathbf{d} - \bar{\mathbf{u}})^{\mathrm{T}}\Sigma^{-1}(\mathbf{d} - \bar{\mathbf{u}})/\sigma^2$. Therefore, when $\theta^2 = 0$, ϕ follows an F distribution with $m - 1$ and $n - p - 1$ degrees of freedom, independent of σ^2. Hence, if $\phi > F_{\alpha,m-1,n-p-1}$, one can conclude that $\theta^2 > 0$ at the α significance level.

2. The expected value of w^2 is $\theta^2 + \sigma^2(1 + \lambda_1 - \lambda_2)$, with $\lambda_1 = (1/m)\,\Sigma\, h_{ii}$ and $\lambda_2 = \{1/[m(m-1)]\}\,\Sigma\Sigma\, h_{ij} \vee i \neq j$ (where h_{ij} represent the ith,jth diagonal element of the $\mathbf{H}_m$ matrix). Hence, a method of moments estimate of θ^2 is $v^2 = w^2 - s^2(1 + \lambda_1 - \lambda_2)$.

Proof: Note that $w^2 = [1/(m-1)]\mathbf{d}^{\mathrm{T}}\mathbf{A}\mathbf{d}$, where $\mathbf{A}$ represents an interclass correlation matrix whose diagonal elements $a_{ii} = (m-1)/m$ and off-diagonal elements $a_{ij} = -1/m$. Thus,

$$\begin{aligned}\mathrm{E}\{[1/(m-1)]\mathbf{d}^{\mathrm{T}}\mathbf{A}\mathbf{d}\} &= [1/(m-1)]\mathrm{E}\{\mathbf{d}^{\mathrm{T}}\mathbf{A}\mathbf{d}\} \\ &= [1/(m-1)][\mathrm{trace}(\mathbf{A}\boldsymbol{\Psi}) \\ &\quad + \mathbf{d}_0^{\mathrm{T}}\mathbf{A}\mathbf{d}_0]\end{aligned}$$

where $\boldsymbol{\Psi} = [\theta^2\, \mathbf{I}_m + \sigma^2(\mathbf{I}_\mathrm{m} + \mathbf{H}_m)]$ (Graybill, 1976). Now, $\mathbf{d}_0^{\mathrm{T}}\mathbf{A}\mathbf{d}_0 = 0$, and

$$\begin{aligned}\mathrm{trace}(\mathbf{A}\boldsymbol{\Psi}) &= \mathrm{trace}[\theta^2\, \mathbf{A}\mathbf{I}_m + \sigma^2(\mathbf{A}\mathbf{I}_m + \mathbf{A}\mathbf{H}_m)] \\ &= \mathrm{trace}(\theta^2\, \mathbf{A}\mathbf{I}_m) + \mathrm{trace}(\sigma^2\mathbf{A}\mathbf{I}_m) \\ &\quad + \mathrm{trace}(\sigma^2\mathbf{A}\mathbf{H}_m)\end{aligned}$$

Furthermore, $\mathrm{trace}(\theta^2\, \mathbf{A}\mathbf{I}_m) = (m-1)\theta^2$, $\mathrm{trace}(\sigma^2\mathbf{A}\mathbf{I}_m) = (m-1)\sigma^2$, and $\mathrm{trace}(\sigma^2\mathbf{A}\mathbf{H}_m) = \{[(m-1)/m]\Sigma\, h_{ii} - (1/m)\Sigma\Sigma\, h_{ij}\}\sigma^2$. Therefore,

$$\begin{aligned}\mathrm{E}\{w^2\} &= [1/(m-1)]((m-1)\theta^2 + (m-1)\sigma^2 \\ &\quad + \{[(m-1)/m]\Sigma\, h_{\mathrm{ii}} - (1/m)\Sigma\Sigma\, h_{\mathrm{ij}}\}\sigma^2) \\ &= \theta^2 + \sigma^2(1 + (1/m)\Sigma\, h_{\mathrm{ii}} \\ &\quad - \{1/[m(m-1)]\}\Sigma\Sigma\, h_{\mathrm{ij}}) \\ &= \theta^2 + \sigma^2(1 + \lambda_1 - \lambda_2)\end{aligned}$$

3. An approximate t-test for $d_0 = 0$ can be computed as $c = \bar{u}/g$, where $g^2 = (1/m)v^2 + 2s^2[(1/m) + h_{mu}]$, $h_{mu} = \mathbf{x}_{mu}^{\mathrm{T}}(\mathbf{X}_n^{\mathrm{T}}\mathbf{X}_n)^{-1}\mathbf{x}_{mu}$, $\mathbf{x}_{mu} = (1/m)(\mathbf{x}_1 + \mathbf{x}_2 + \ldots + \mathbf{x}_m)$, and c is compared with a t distribution with $n - p - 1$ degrees of freedom.

Proof: Define $\mathbf{y}_2$ as the vector of m observed samples from the second time frame, $\hat{\mathbf{y}}_1$ as the corresponding vector of predicted values for these m samples for the first time frame, $\mathbf{y}_1$ as the corresponding vector of m (unobserved) samples from the first time frame, and let $\mathbf{q}$ represent a vector of length m, where $\mathbf{q}$ is defined as $\mathbf{q} = (1/m, 1/m, \ldots, 1/m)$. Clearly, we need to derive the sampling distribution of $\mathbf{q}^T(\mathbf{y}_2 - \mathbf{y}_1)$. Now, note that $\mathbf{q}^T(\mathbf{y}_2 - \mathbf{y}_1) = \mathbf{q}^T[(\mathbf{y}_2 - \hat{\mathbf{y}}_1) - (\mathbf{y}_1 - \hat{\mathbf{y}}_1)]$ and that conditional on $\mathbf{X}_1$, $(\mathbf{y}_2 - \hat{\mathbf{y}}_1) \sim N[\mathbf{d}_0, \theta^2 \mathbf{I}_m + \sigma^2(\mathbf{I}_m + \mathbf{H}_m)]$ and $(\mathbf{y}_1 - \hat{\mathbf{y}}_1) \sim N[\mathbf{0}, \sigma^2(\mathbf{I}_m + \mathbf{H}_m)]$. Furthermore, these distributions are independent under our modeling assumptions. Hence, $\mathbf{q}^T(\mathbf{y}_2 - \mathbf{y}_1) \sim N\{\mathbf{q}^T\mathbf{d}_0, \mathbf{q}^T[\theta^2 \mathbf{I}_m + 2\sigma^2(\mathbf{I}_m + \mathbf{H}_m)]\mathbf{q}\}$. Now, note that $\mathbf{q}^T\mathbf{d}_0 = d_0$ and that $\mathbf{q}^T[\theta^2 \mathbf{I}_m + 2\sigma^2(\mathbf{I}_m + \mathbf{H}_m)]\mathbf{q} = \theta^2/m + 2\sigma^2/m + \sigma^2(\mathbf{q}^T\mathbf{H}_m\mathbf{q})$, and that $\mathbf{q}^T\mathbf{H}_m\mathbf{q} = h_{mu}$, where $h_{mu} = \mathbf{x}_{mu}^T(\mathbf{X}_n^T\mathbf{X}_n)^{-1}\mathbf{x}_{mu}$, with $\mathbf{x}_{mu} = (1/m)(\mathbf{x}_1 + \mathbf{x}_2 + ... \mathbf{x}_m)$. Therefore, since $E(\bar{u} \mid \mathbf{X}_1) = d_0$, the quantity $(\bar{u} - d_0)/[\theta^2/m + 2\sigma^2(1/m + h_{mu})]^{0.5}$ would be distributed as a $N(0,1)$ random variable, if both θ^2 and σ^2 were known. Likewise, this quantity would be distributed as a t random variable with $n - p - 1$ degrees of freedom if θ^2 was known and s^2 was used as an estimate of σ^2. In practice, θ^2 will also be estimated (using v^2), and hence the quantity $\bar{u}/g$, where $g^2 = (1/m)v^2 + 2s^2[(1/m) + h_{mu}]$ will in general only have an approximate t distribution.

This test statistic assumes that the second set of m samples are not collocated with the first set of n sites. When the two sets of sample sites are perfectly collocated, then the distribution of $(\mathbf{y}_1 - \hat{\mathbf{y}}_1)$ becomes $(\mathbf{y}_1 - \hat{\mathbf{y}}_1) \sim N[\mathbf{0}, \sigma^2(\mathbf{I}_m - \mathbf{H}_m)]$. Hence, the $\mathbf{H}_m$ hat matrix drops out of the variance estimate, and $g^2 = (1/m)(v^2 + 2s^2)$. Therefore, the test shown in Result 4 above will tend to be slightly conservative if some of the sample sites from the second time frame coincide with sites from the first time frame.

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